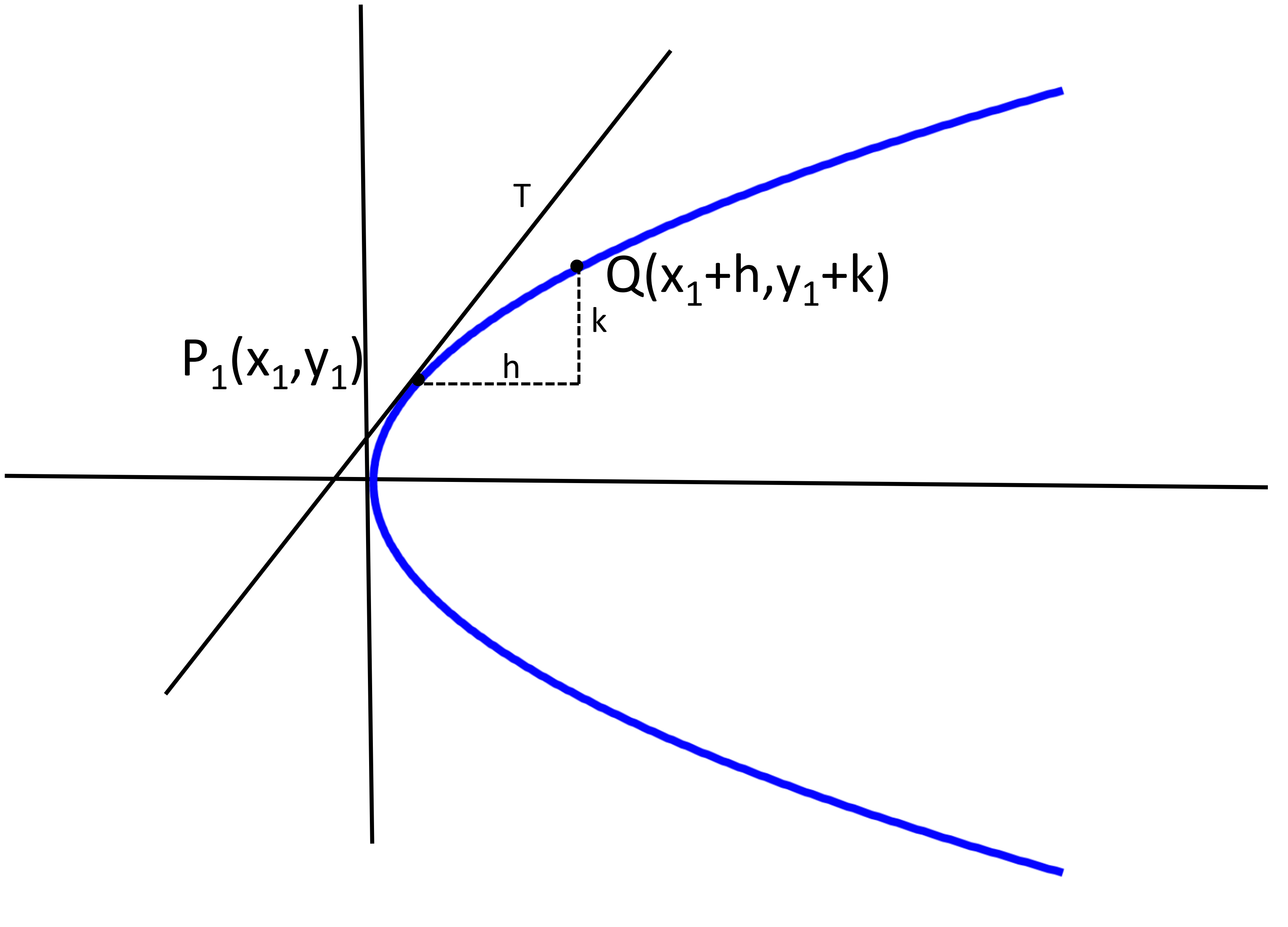
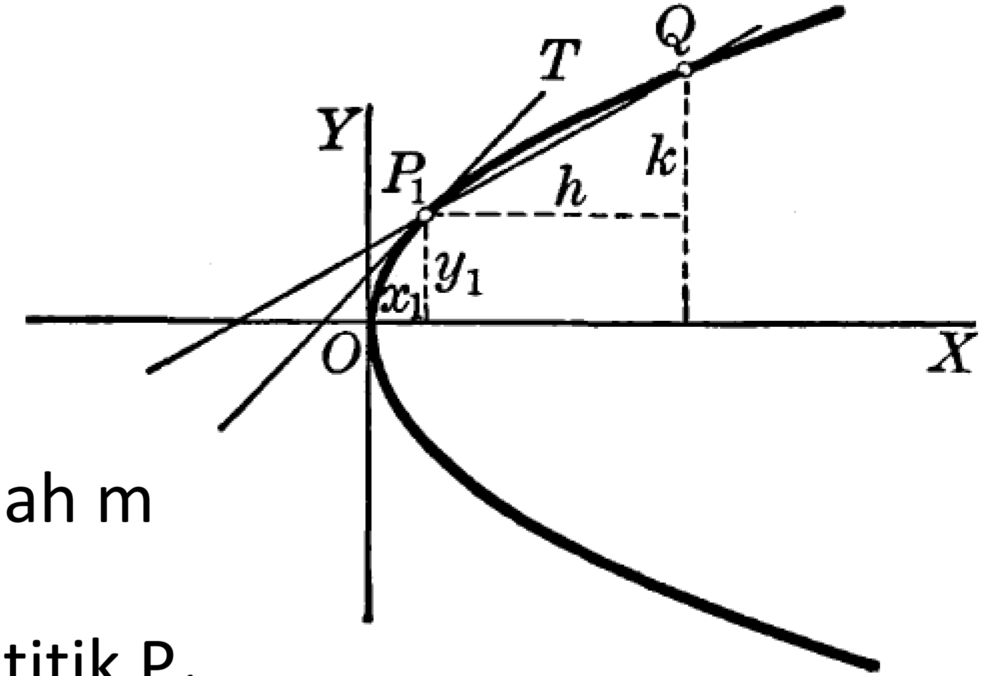
ALTERNATIF MENENTUKAN GARIS SINGGUNG PADA IRISAN KERUCUT


T
Q(x1+h,y1+k)
k
h
P1(x1,y1)

maka mestilah kemiringan dari P_1Q adalah k/h



kemiringan garis P_1Q adalah m

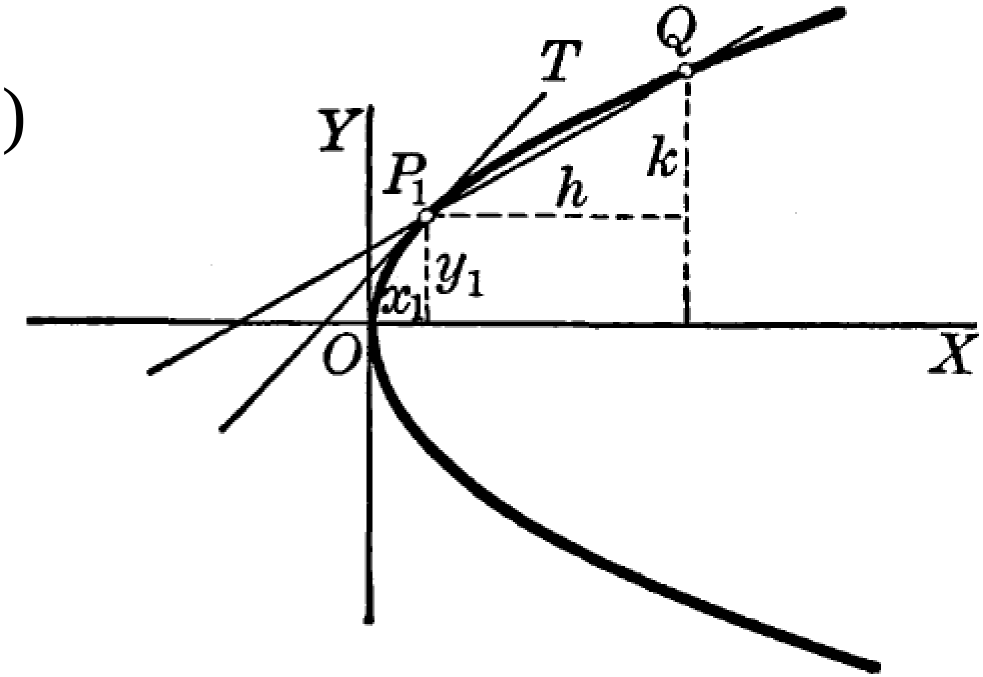
Apabila titik Q mendekati titik P_1

$$m = \lim_{Q \to P_1} \frac{k}{h}$$

Karena P_1 dan Q keduanya berada pada parabola, maka berlaku

$$y_1^2 = 4px_1 \qquad (y_1 + k)^2 = 4p(x_{1+h})$$

Dari persamaan (1) dan (2) diperoleh



$$k(2y_1 + k) = 4ph$$

$$\frac{k}{h} = \frac{4p}{2y_1 + k}$$

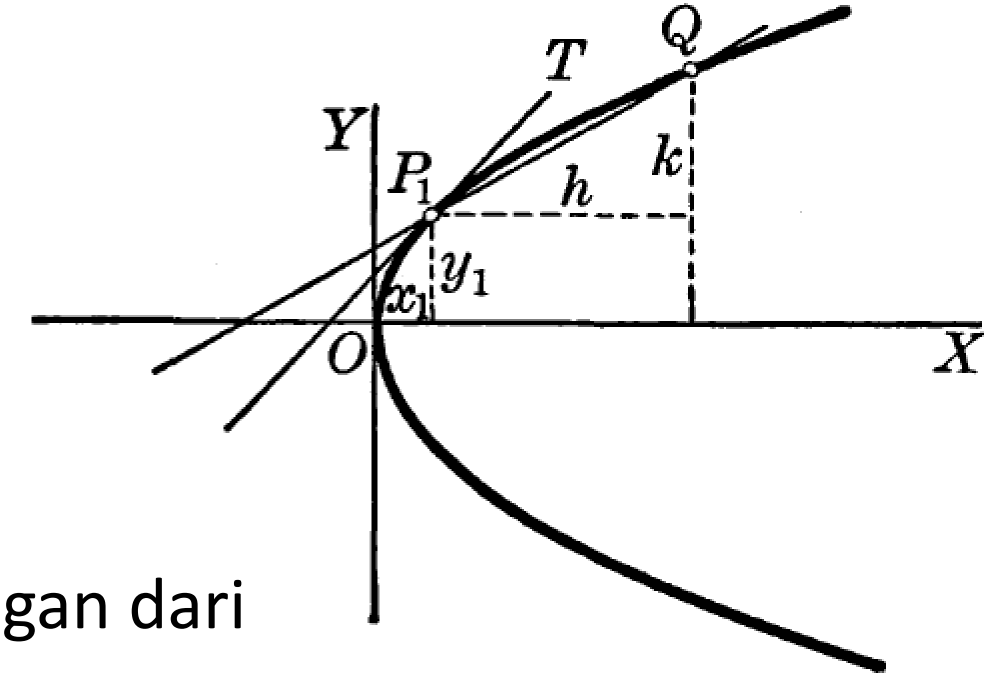
Apabila $Q \rightarrow P_1$ maka $k \rightarrow 0$, sehingga

$$\lim \frac{k}{h} = \frac{4p}{2y_1}$$

dipihak lain $\lim \frac{k}{h} = m,$

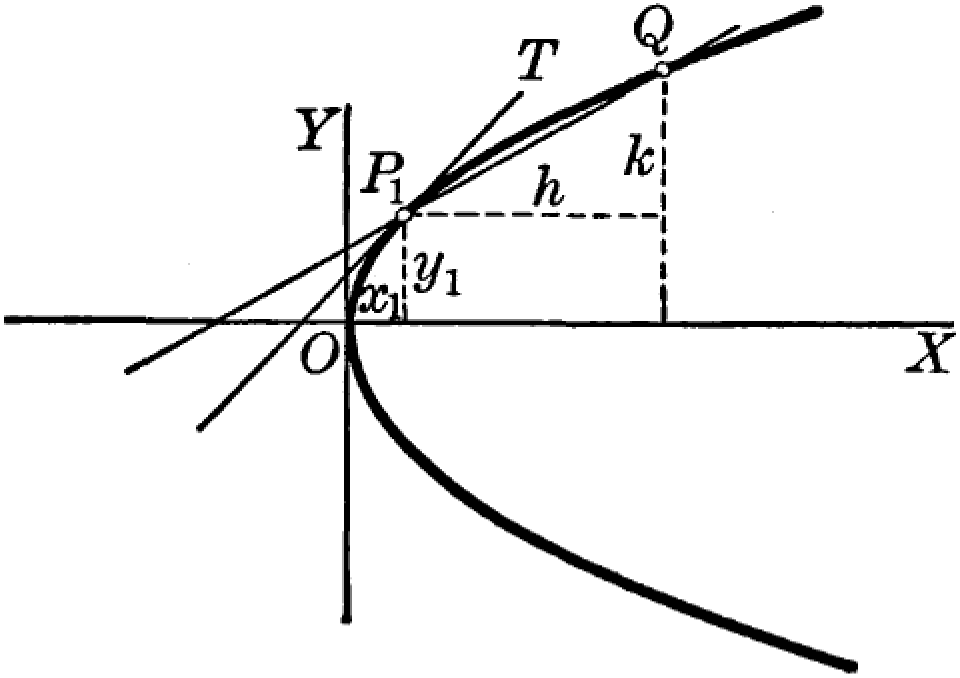
Yang merupakan kemirin
dari P_1T. jadi diperoleh

$$m = \frac{2p}{y_1}$$



yang merupakan kemiringan dari persamaan garis P_1T yaitu kemiringan garis singgung dari titik $P_1(x_1,y_1)$ pada parabola $y^2 = 4px$. Sehingga kalau kita ingin menentukan persamaan garis singgungnya, dapat digunakan persamaan garis melalui titik $P_1(x_1,y_1)$ dengan gradient $m = \frac{2p}{y_1}$

$$y - y_1 = \frac{2p}{y_1}(x - x_1)$$

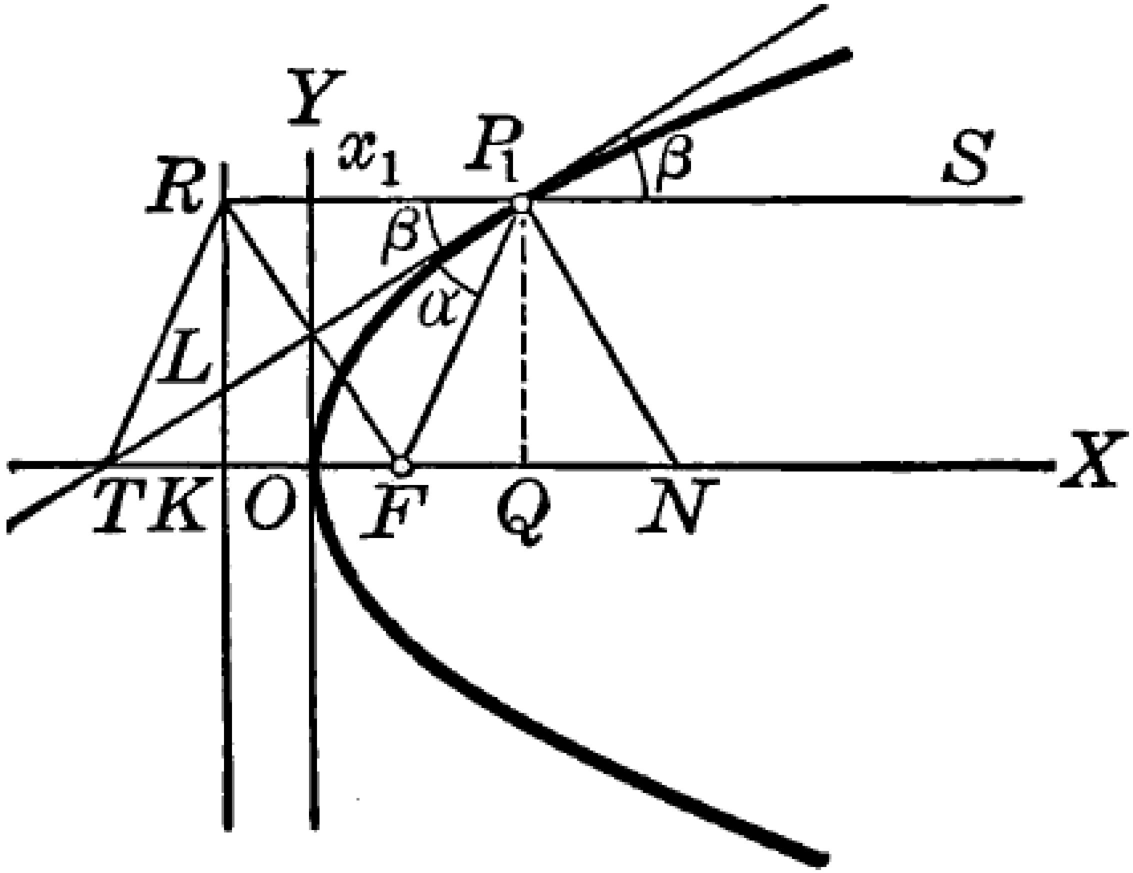


$$y_1 y - y_1^2 = 2px - 2px_1$$

Karena $y_1^2 = 4px_1$

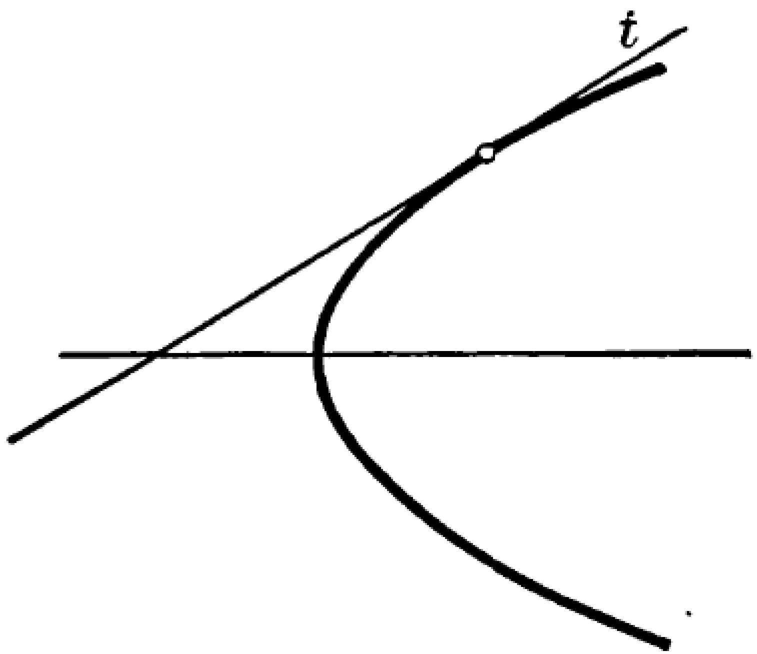
$$y_1 y = 2p(x - x_1)$$

Properties of the Parabola.



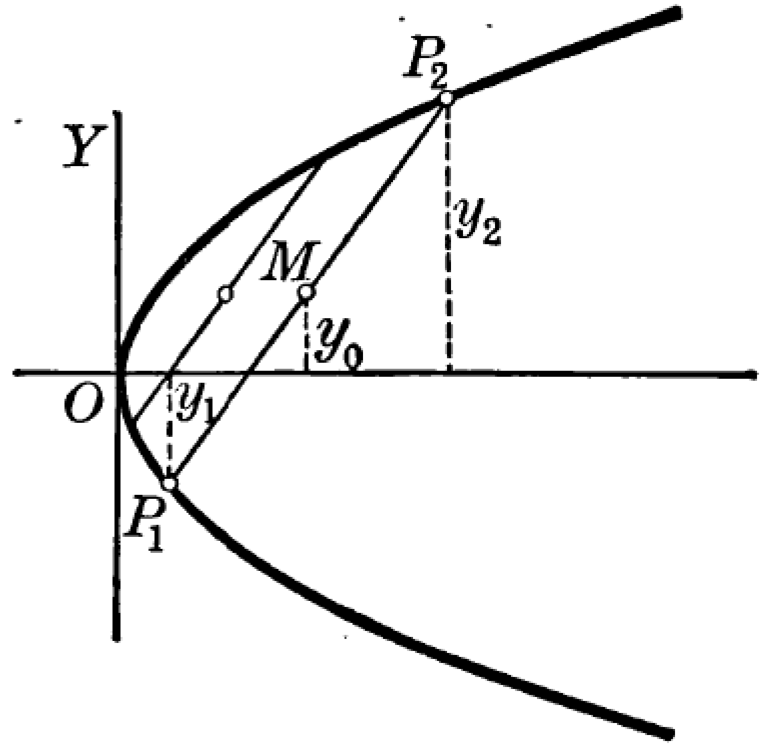
Siapa yang minat membahas ini saya sediakan buku dan jurnalnya

131. *To find the equation of the line which has the slope m and is tangent to the parabola $y^2 = 4px$.*



Yang ini memang baik dengan subsitusi

Misalkan parabolanya adalah $y^2 = 4px$ dengan gradient garis adalah m, untuk itu misalkan persamaan garis yang melalui P_1P_2 adalah y = mx + n. kalau persamaan garis kita subsitusikan pada parabola maka akan diperoleh



$$my^2 - 4py + 4pn = 0$$

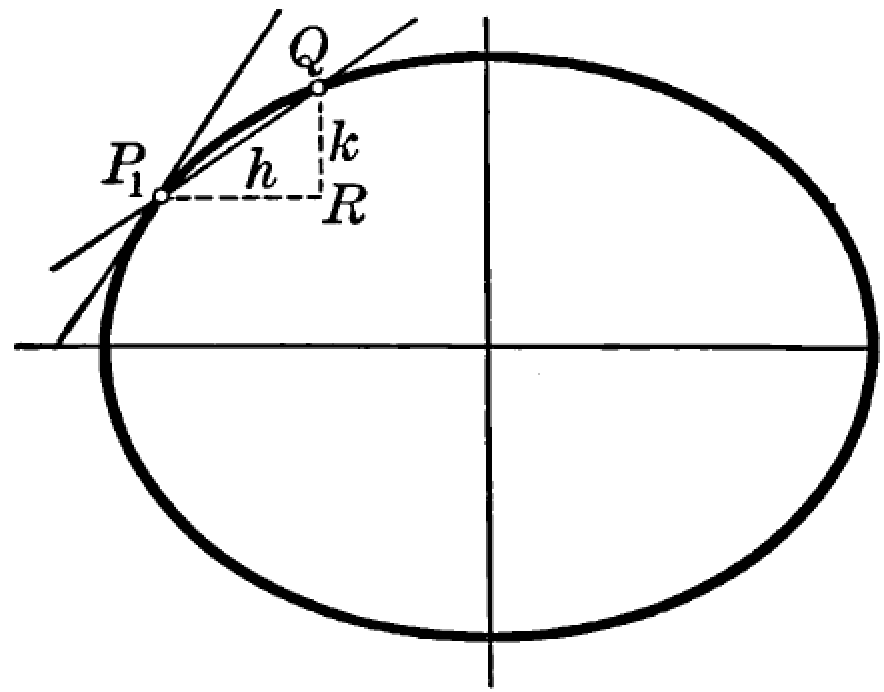
Diperoleh y_1 dan y_2

M adalah titik tengah dari P_1P_2

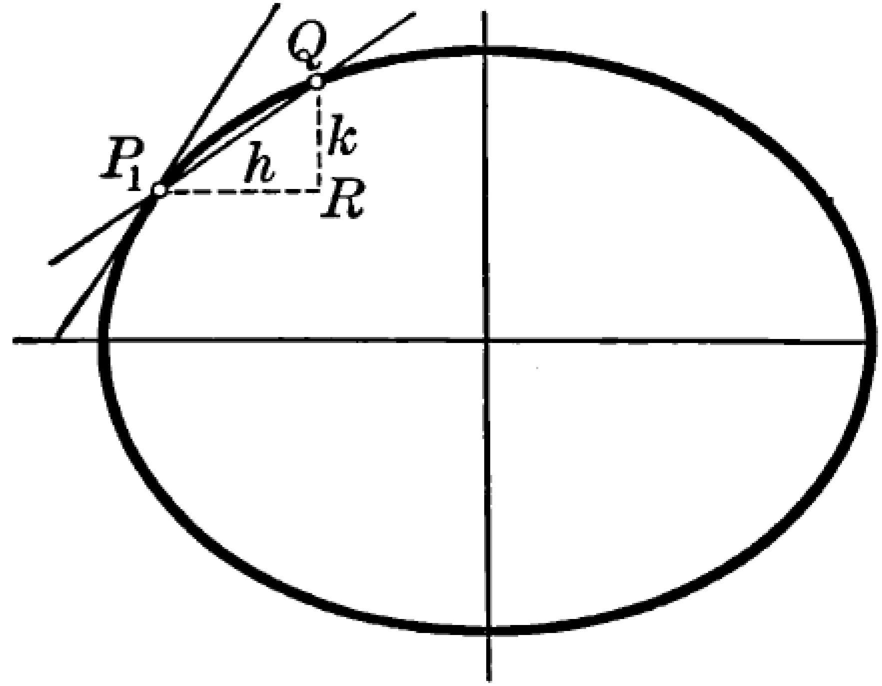
$$y_0 = \tfrac{1}{2}(y_1 + y_2)$$

$$= \tfrac{1}{2}(4p/m) = 2p/m$$

147. *To find the slope of the ellipse* $x^2/a^2 + y^2/b^2 = 1$ *at any point* $P_1(x_1, y_1)$ *on the ellipse.*



Let a second point on the given ellipse be

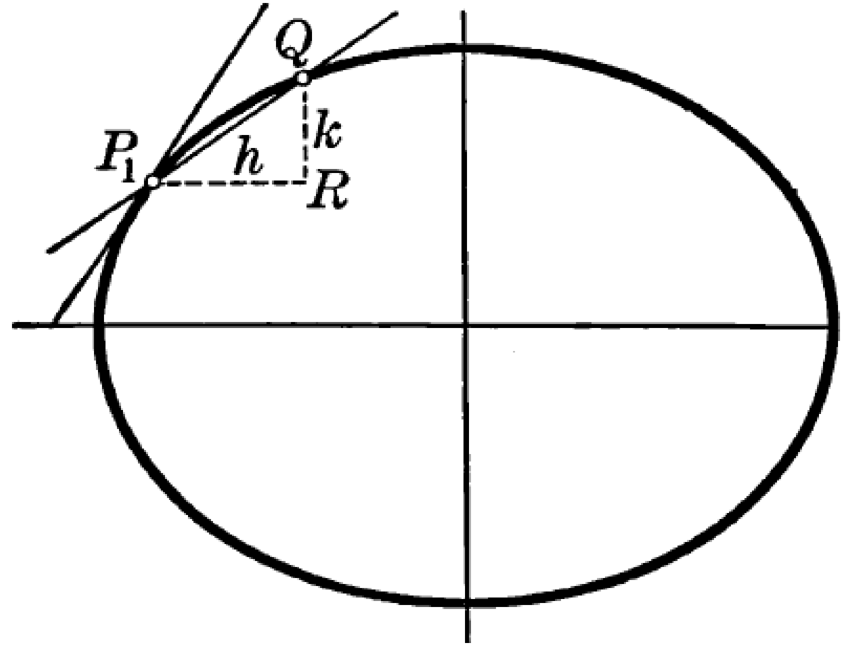


$Q(x_1 + h,\ y_1 + k)$, where $PR = h,\ RQ = k$.

Then the slope of the secant P_1Q is k/h.

Since the points $P_1(x_1, y_1)$ and $Q(x_1 + h,\ y_1 + k)$ are on the ellipse $b^2x^2 + a^2y^2 = a^2b^2$,

we have



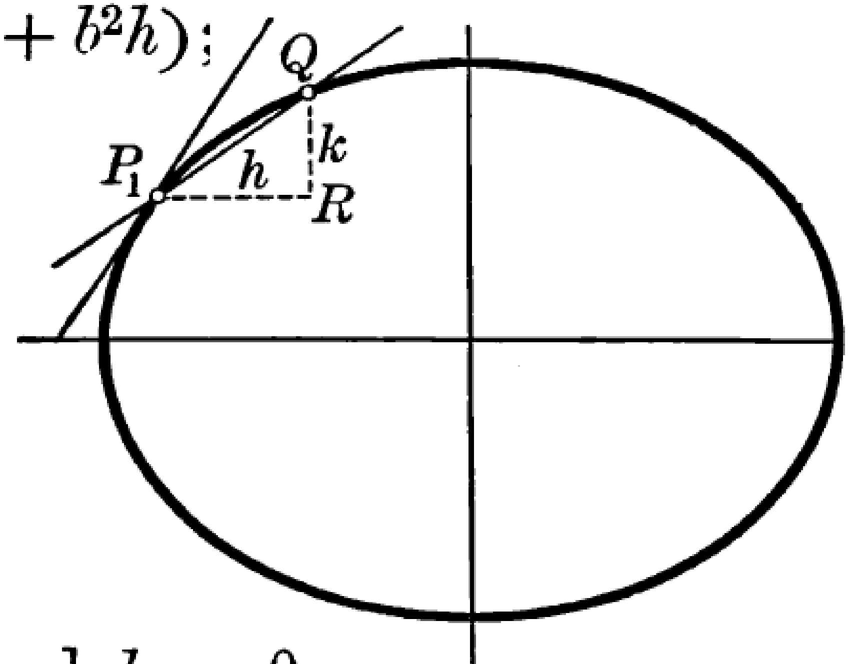
$$b^2x_1^2 + a^2y_1^2 = a^2b^2, \qquad (1)$$

$$b^2(x_1 + h)^2 + a^2(y_1 + k)^2 = a^2b^2. \qquad (2)$$

Subtracting (1) from (2), we have

$$k(2\,a^2y_1 + a^2k) = -\,h(2\,b^2x_1 + b^2h);$$

$$k(2a^2y_1 + a^2k) = -h(2b^2x_1 + b^2h);$$



$$\frac{k}{h} = -\frac{2b^2x_1 + b^2h}{2a^2y_1 + a^2k}.$$

Now when $Q \to P_1$, $h \to 0$ and $k \to 0$;

$$\lim k/h = -2b^2x_1/2a^2y_1.$$

But $\lim k/h = m$,

the slope of the tangent at $P_1(x_1, y_1)$.

$$\boldsymbol{m = -\frac{b^2x_1}{a^2u.}},$$

which is also the slope of the ellipse at $P_1(x_1, y_1)$.

To find the equation of the tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ *at the point* $P_1(x_1, y_1)$.

$$y - y_1 = -\frac{b^2 x_1}{a^2 y_1}(x - x_1);$$

$$b^2 x_1 x + a^2 y_1 y = b^2 x_1^2 + a^2 y_1^2.$$

Since (x_1, y_1) is on the ellipse,

we have $b^2 x_1^2 + a^2 y_1^2 = a^2 b^2$.

Hence

$$b^2 x_1 x + a^2 y_1 y = a^2 b^2,$$

$$\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1.$$

149. Corollary 1. *The equation of the normal to the ellipse* $x^2/a^2 + y^2/b^2 = 1$ *at the point* $P_1(x_1, y_1)$ *is*

$$y - y_1 = \frac{a^2 y_1}{b^2 x_1}(x - x_1).$$

150. Corollary 2. *The intercepts of the tangent and normal at the point* $P_1(x_1, y_1)$ *on an ellipse are as follows:*

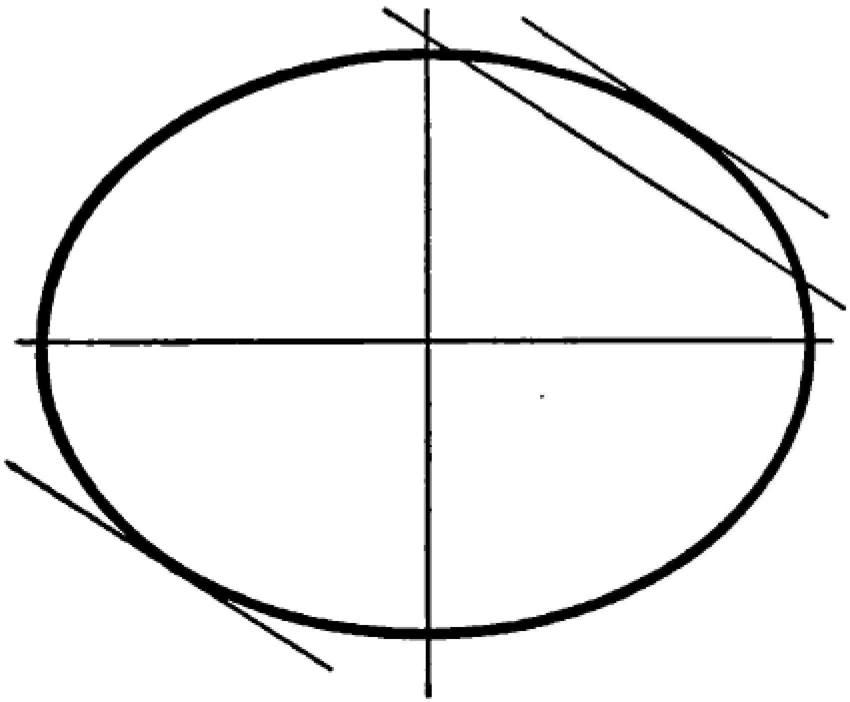
1. x intercept of tangent, $x = \dfrac{a^2}{x_1};$

2. y intercept of tangent, $y = \dfrac{b^2}{y_1};$

3. x intercept of normal, $$x = \frac{a^2 - b^2}{a^2} x_1 = e^2 x_1;$$

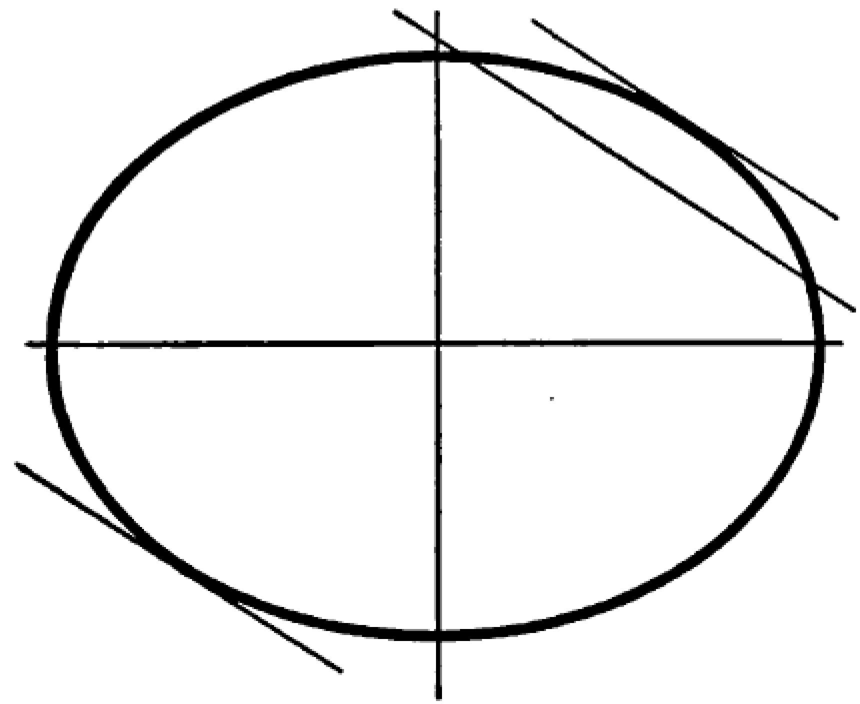
4. y intercept of normal, $$y = \frac{b^2 - a^2}{b^2} y_1.$$

151. *To find the equations of the lines which have the slope m and are tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$.*



$$y = mx \pm \sqrt{a^2m^2 + b^2}.$$

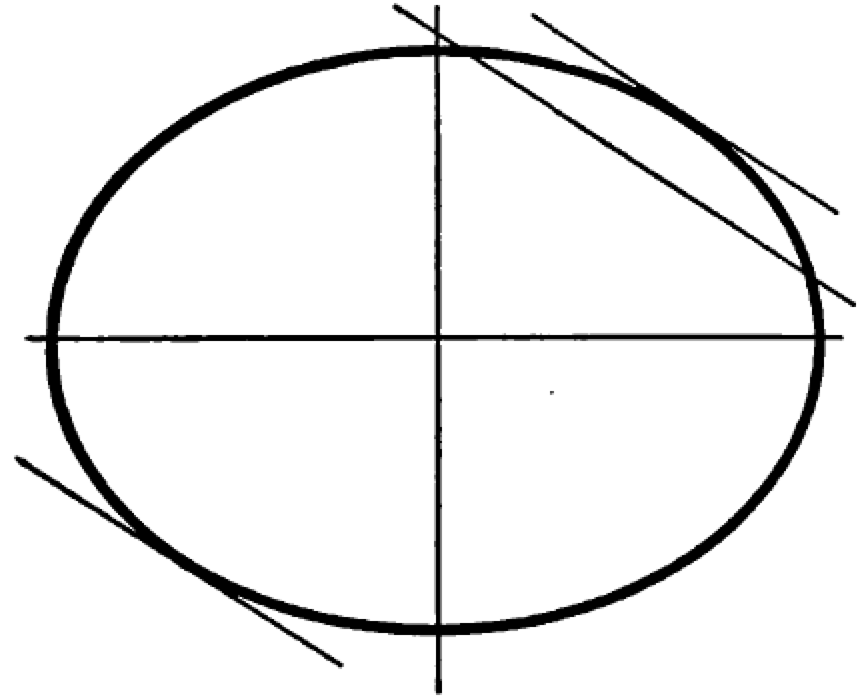
151. *To find the equations of the lines which have the slope m and are tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$.*



Solution. Let the equation $y = mx + k$ represent any line having the slope m. To find the common points of this line and the ellipse we regard their equations as simultaneous. Thus, the equation of the ellipse being

$$b^2x^2 + a^2y^2 = a^2b^2,$$

$$b^2x^2 + a^2(mx + k)^2 = a^2b^2,$$



$$(b^2 + a^2m^2)\,x^2 + 2\,a^2mkx + a^2(k^2 - b^2) = 0.$$

$$(2\,a^2mk)^2 - 4\,(b^2 + a^2m^2)\,a^2(k^2 - b^2) = 0\,;$$

$$k = \pm\sqrt{a^2m^2 + b^2}.$$

Hence there are two tangents having the slope m, namely,

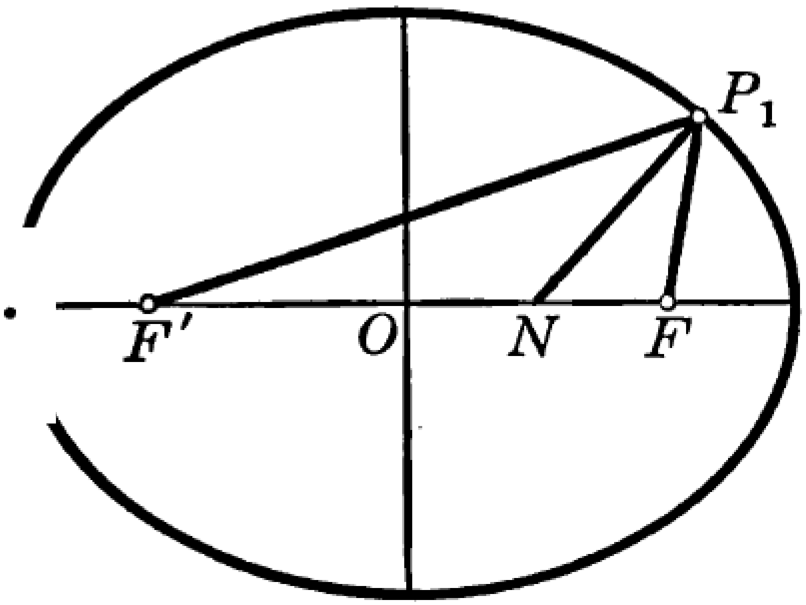
$$y = mx \pm \sqrt{a^2m^2 + b^2}.$$

152. *The normal at any point $P_1(x_1, y_1)$ on an ellipse bisects the angle between the focal radii of the point P_1.*

Since $$F'O = OF = ae,$$

and the x intercept

$$= \frac{a + ex_1}{a - ex_1}.$$



of the normal is ON, where

$$ON = e^2x_1,$$

we have $$\frac{F'N}{NF} = \frac{F'O + ON}{OF - ON} = \frac{ae + e^2x_1}{ae - e^2x_1} = \frac{a + ex_1}{a - ex_1}.$$

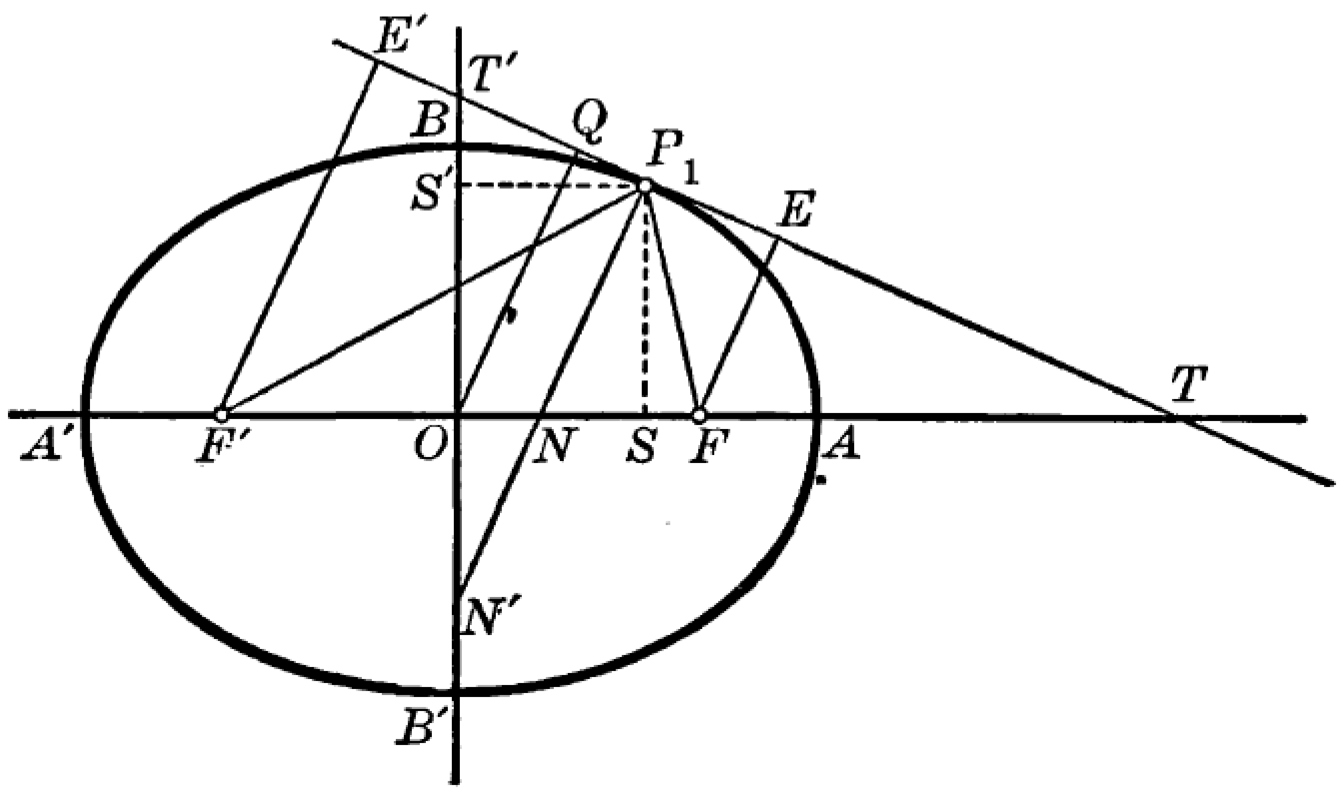
Since in proving the theorem of § 143 we showed that $F'P_1 = a + ex_1$ and $FP_1 = a - ex_1$, we have

$$\frac{F'N}{NF} = \frac{F'P_1}{FP_1}.$$

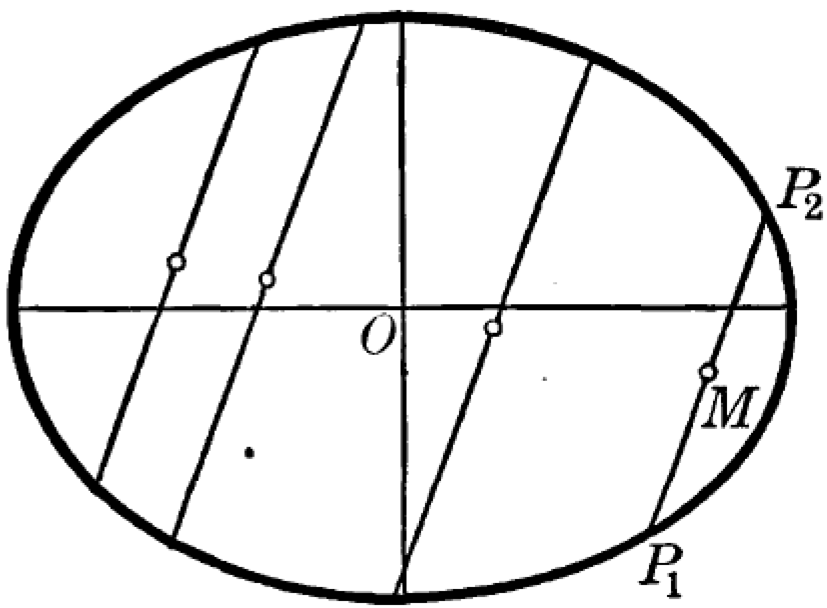
Therefore, since P_1N divides $F'F$ into segments proportional to $F'P_1$ and FP_1, P_1N bisects the angle $F'P_1F$.

Properties of the Ellipse

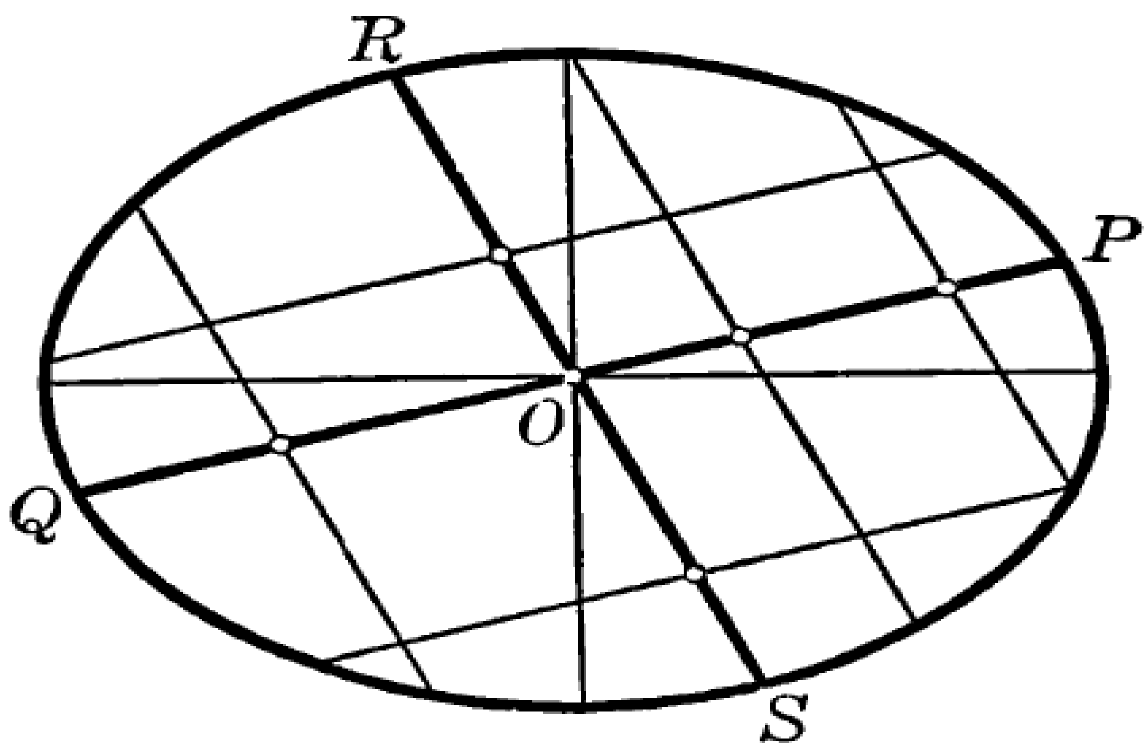
1. The axes of an ellipse are normal to the ellipse, and no other normal passes through the center.



153. *To find the locus of the mid points of a system of parallel chords of an ellipse.*



155. *If one diameter of an ellipse bisects the chords parallel to another diameter, the second diameter bisects the chords parallel to the first.*



Tentukan persamaan garis singgung pada hiperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

1. Jika gradiennya adalah m
2. Disebarang titik $P_1(x_1,y_1)$